FE - SEM II (CBGS)
Applied maths - II

May 2014
15/5/14

(REVISED COURSE) QP Code : **NP-17762**

(3 Hours) **[Total Marks : 80**

N.B. : (1) Question **No.1** is **compulsory.**
(2) Attempt any **three** question from remaining **five** questions.

1. (a) Evaluate $\int_0^1 \sqrt{(\sqrt{x}-x)}\,dx$ **3**

(b) Solve $[D^4 - 4D^3 + 8D^2 - 8D + 4]\,y = 0$ **3**

(c) Prove that $(1+\Delta)(1-\nabla) = 1$ **3**

(d) Change to polar co-ordinate and evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2+y^2)\,dy\,dx$ **3**

(e) Solve $(x^2 - 4xy - 2y^2)\,dx + (y^2 - 4xy - 2x^2)\,dy = 0$ **4**

(f) Evaluate $\int_0^a \int_0^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2}\,dy\,dx$ **4**

2. (a) Solve $xy(1+xy^2)\frac{dy}{dx} = 1$ **6**

(b) Change the order of integration and evaluate $\int_0^\infty \int_0^x x e^{-x^2/y}\,dy\,dx$ **6**

(c) Evaluate $\int_0^{\pi/2} \frac{dx}{a^2\sin^2 x + b^2\cos^2 x}$ and show that **8**

$$\int_0^{\pi/2} \frac{dx}{\left(a^2\sin^2 x + b^2\cos^2 x\right)^2} = \frac{\pi}{4ab}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)$$

3. (a) Evaluate $\iiint x^2 yz\,dx\,dy\,dz$ throughout the volume bounded by $x = 0, y = 0, z = 0,$ $x + y + z = 1.$ **6**

(b) Find the area bounded by parabola $y^2 = 4x$ and the line $y = 2x - 4$. **6**

(c) Use the method of variation of parameter to solve **8**

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x}\sec^2 x\,(1 + 2\tan x)$$

4. (a) Find the total length of the loop of the curve $9y^2 = (x+7)(x+4)^2$. **6**

(b) Solve $\frac{d^2y}{dx^2} + 2y = x^2e^{3x} + e^x - \cos 2x$ **6**

(c) Apply Runge-kutta method of fourth order to find an approximate value of y at x = 0.2 if $\frac{dy}{dx} = x + y^2$ give that y = 1, when x = 0 in step of h = 0.1. **8**

5. (a) Solve $y(x^2y + e^x)\,dx - e^x\,dy = 0$. **6**

(b) Using Taylor's series method solve $\frac{dy}{dx} = 1 - 2xy$ given that y(0) = 0 and hence find y(0.2) and y(0.4). **6**

(c) Compute the value of the definite integral $\int_{0.2}^{1.4} \left(\sin x - \log_e x + e^x\right) dx$, by **8**

 (i) Trapezoidal Rule
 (ii) Simpson's one third Rule
 (iii) Simpson's three-eigth Rule.

6. (a) The motion of a particle is given by $\frac{d^2x}{dt^2} = -k^2x - 2h\frac{dx}{dt}$, solve the equation when h = 5, k = 4 taking x = 0, $v = v_0$ at t = 0. Show that the time of maximum displacement is independent of the initial velocity. **6**

(b) Evaluate $\iint \left(x^2 + y^2\right) dx\,dy$ over the area of triangle whose vertices are (0, 0), (1, 0), (1, 2). **6**

(c) Find the volume bounded by $y^2 = x$, $x^2 = y$ and the planes z = 0 and x + y + z = 1. **8**